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TITLE   Monte Carlo Approaches to Light Nuclei:
Structure and Electron Scattering

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MONTE CARLO APPROACHES TO LIGHT NUCLEI:
Structure and Electron Scattering

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Significant progress has been made recently in the application of Monte Carlo methods to the study of light nuclei. We review new Green's function Monte Carlo results for the alpha particle, Variational Monte Carlo studies of ^{16}O, and methods for low-energy scattering and transitions. Through these calculations, a coherent picture of the structure and electromagnetic properties of light nuclei has arisen. In particular, we examine the effect of the three-nucleon interaction and the importance of exchange currents in a variety of experimentally measured properties, including form factors and capture cross sections.

1. INTRODUCTION

Few- and many-body problems in nuclear systems have a long history, but only in recent times have the computational techniques and facilities been adequate to fully attack these problems. The difficulties are primarily due to the strong correlations arising from the nuclear force, correlations which require relatively sophisticated algorithms. One important success story is the development of Faddeev methods for A=3. Another, which we will review in this article, is the application of Monte Carlo methods to light nuclei.

We attempt, in these calculations, to solve the non-relativistic Schroedinger equation:

$$H|\Psi\rangle = \left[\sum_i -\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + ...\right]\Psi = E|\Psi\rangle \qquad (1)$$

for a Hamiltonian determined by fitting two- (and possibly three-) body experimental data. Clearly such a non-relativistic treatment is only a first approximation; nevertheless a great deal of physics can be examined in this way.

We will review recent developments in Monte Carlo techniques as applied to nuclei, including Variational[1] (VMC) and Green's Function Monte Carlo[2,3] (GFMC) methods, which have proven to be very valuable in studying light nuclei. These methods have for the most part originally been developed in condensed matter physics, where they have been used to study quantum fluids and solids.[4,5] The Hamiltonian in these systems is at least superficially similar to that in nuclear physics, it consists of a strong short-range repulsion and an attractive force at larger distances.

The Hamiltonian in nuclear physics, though, is complicated by the strong spin-isospin dependence of the interaction. We will concentrate chiefly on the Argonne[6] NN interaction,

which may be written:

$$V_{ij} = \sum_{i<j} V^k(r_{ij})O^k_{ij} \qquad (2)$$

where the operators O^k are

$$O^k_{ij} = 1, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S_{ij}, L \cdot S^2_{ij}, L^2_{ij} \qquad (3)$$

multiplied by either an isospin-independent (1) or -dependent ($\tau_i \cdot \tau_j$) operator. All modern interactions (Argonne,[6] Bonn,[7] Nijmegen[8] ...) may be written in a similar manner, although the choice of non-local operators varies. These interactions consist of a one-pion interaction at long distances, an intermediate range attraction, and a short-range phenomenological repulsion, and are fit to the deuteron as well as two-body scattering data.

In a similar spirit, the three-nucleon-interaction (TNI) at long distances is assumed to have the structure of a two-pion-exchange interaction, but its precise strength is adjusted to fit the three-body binding energy.[9] The full TNI consists of the two-pion exchange piece $V'_{2\pi}$ and a short-range repulsive term:

$$V_{ijk} = U_0 \sum_{cyc} W_{2\pi}(r_{ij})W_{2\pi}(r_{ik}) + A_0 \sum_{cyc} V_{2\pi}(\vec{r}_{ij}, \vec{r}_{ik}), \qquad (4)$$

where the sums run over cyclic permutations of the particles, and the function $W_{2\pi}$ has the range of a two-pion interaction. The parameters U_0 and A_0 can be estimated from calculating the effects of suppressing Δ degrees of freedom, but their precise values are determined by fitting the binding energy of A=3 nuclei. The three-body force is quite small compared to the two-nucleon interaction, but nevertheless provides an important fraction of the total binding energy.

The Monte Carlo methods (Variational and Green's Function Monte Carlo) used to solve the Schroedinger equation are presented in the next section. We then review a variety of recent results concerning the structure and electromagnetic properties of light nuclei. In particular, we examine the question of three-nucleon-interactions, the importance of the tensor interaction, and electromagnetic form factors and transitions. Exchange currents will be seen to have a decisive role in many electromagnetic properties.

2. Monte Carlo Methods

Variational Monte Carlo (VMC) is a variational method often employed to study the ground state and low-lying excitations of quantum systems. A generalized Jastrow form is assumed for the wave function:

$$|\Psi\rangle = S \prod_{i<j} F_{ij}|\Phi\rangle, \qquad (5)$$

and the expectation value of the Hamiltonian is minimized with respect to variational pa rameters in the wave function. In this equation, Φ is an anti-symmetric Slater determinant

of one-particle states, and the F_{ij} are pair correlation operators:

$$F_{ij} = f^c(r_{ij}) \left[1 + u_3 \left(\sum_k u^k(r_{ij}) O_{ij}^k \right) \right] \tag{6}$$

which include the most important spin-isospin operators in the Hamiltonian. The operators for different pairs do not commute, so we introduce the symmetrization operator S to obtain an overall anti-symmetric wave function. The pair correlations are obtained by solving two-body differential equations of the general form:

$$[-\frac{\hbar^2}{m}\nabla^2 + v(r) + \lambda(r)]F = 0, \tag{7}$$

where the function λ contains several variational parameters. The u_3 correlation in equation 6 is a three-body term which reduces the strength of the operator-dependent two-body correlations for some configurations of the nucleons.[1] The complete wave function Ψ is constructed to have the correct asymptotic properties as one nucleon is separated from the system.

The straightforward variational Monte Carlo algorithm is limited to treating small systems, optimistically up to A $\approx$ 8. For the spin-independent interactions in condensed matter physics, it is possible to simulate one to two hundred particles. For the interactions of interest in nuclear physics, however, the problems are much more complex. The wave function of a nucleus consists of $2^A \frac{A!}{N!Z!}$ spin-isospin components, the first factor represents the spin (up or down for each nucleon) and the second the isospin. These states are explicitly summed in light nuclei.

In order to treat larger systems, however, another method must be developed to perform this sum. In principle, this could be done with Monte Carlo, but schemes employed to date yield fairly high variance. Pieper, et al.[10] have recently introduced a cluster approximation scheme in an attempt to overcome this problem. In this method, the expectation value of an operator O is written as a sum over N-body clusters, where N ranges from 1 to A. In an N-body cluster, only the spin-dependent correlation operators which act within the clusters are taken into account. For example, if we write the correlation operator F_{ij} (Eq. 6) as $f_{ij}^c[1 + U_{ij}]$, the two body cluster approximation to the potential energy V_{ij} is:

$$V_{ij} = \frac{\langle \Phi | (\prod_{k<l} f_{kl}^c)[1 + U_{ij}]^\dagger V_{ij}[1 + U_{ij}](\prod_{k<l} f_{kl}^c)\Phi \rangle}{\langle \Phi | (\prod_{k<l} f_{kl}^c)[1 + U_{ij}]^\dagger [1 + U_{ij}](\prod_{k<l} f_{kl}^c)\Phi \rangle}. \tag{8}$$

The full spin-independent jastrow wave function is taken into account at each step. Three-body clusters involve expressions of the form $([1 + U_{ij}][1 + U_{ik}][1 + U_{jk}])^\dagger V_{ij}[1 + U_{ij}][1 + U_{ik}][1 + U_{jk}]$, and so on. With current techniques, it is possible to analyze up to four-body clusters. The calculation is only strictly variational if up to A-body clusters are taken into account, but results to date show very good convergence for most expectation values, indicating that keeping a smaller subset is a good approximation. Current methods work well for oxygen, but are still limited by an increasing statistical error for larger systems.

This specific form (Eq. 5) of variational wave function is adequate for many purposes, yielding ground state energies within a few per cent of the Faddeev values for $A=3$. It also gives very similar results for the electromagnetic form factors.[11] Further improvements are possible by including $L \cdot S$ correlations and three-body terms.[12] It is necessary, though, to develop exact methods to provide adequate tests of these wave functions for $A \geq 4$.

GFMC methods project out the ground state of a quantum system through:

$$|\Psi_0) = \lim_{r \to \infty} \exp(-Hr)|\Psi_T), \tag{9}$$

where $|\Psi_T)$ is an initial trial state typically obtained from a variational calculation. In general one cannot compute $\exp(-Hr)$, but by dividing the propagation time r into many small steps Δr,

$$\exp(-Hr) = \prod_1^n \exp(-H\Delta r) = \int G(\vec{R}_n, \vec{R}_{n-1})....G(\vec{R}_1, \vec{R}_0) \tag{10}$$

the full propagator can be evaluated by Monte Carlo. In practice, one must use several time steps Δr and extrapolate to $\Delta r = 0$ in order to eliminate time step errors associated with the non-commuting nature of the kinetic and potential terms. The fact that the potential acting between different pairs does not commute is an important aspect of nuclear physics problems which makes it difficult to use more accurate analytic methods to approximate the pair Green's function.

For short propagation times Δr and static potentials, the following approximation[14] to the propagator is quite useful:

$$G(\vec{R}, \vec{R}') \approx G^0(\vec{R}, \vec{R}') \prod_{i<j} \frac{g_{ij}(\vec{r}_{ij}, \vec{r}'_{ij})}{g_{ij}^0(\vec{r}_{ij}, \vec{r}'_{ij})}. \tag{11}$$

In this equation, the full G for 3A coordinates is approximately given by the free particle propagator (a gaussian) times a product of all pair propagators divided by their respective free particle propagators. The simplest approximation to the ratio in equation 11 is,

$$g_{ij}/g_{ij}^0 = (\exp[-(\Delta r/2)(V_{ij}(r) + V_{ij}(r'))], \tag{12}$$

where V, and consequently g_{ij}, are operators in spin-isospin space. In fact, we perform a further sum over 'sub-paths' of the two particles in order to determine the two-body Green's function.[13] We use time steps on the order of $3 - 5 \times 10^{-3}$ MeV^{-1}, which yield very small extrapolations to zero time step.

Incorporating momentum-dependent terms in the GFMC is more difficult. Realistic models of the NN interaction do contain such pieces, including $L \cdot S, L \cdot S^2, L^2$, and p_{ij}^2 operators. To date, we have only been able to include the first of these operators, $L \cdot S$, successfully in the exact GFMC algorithm. The difficulties in treating the second-order derivatives term are discussed in reference 13, and are essentially due to the fact that the nucleons gain different effective masses in the different spin-isospin channels.

However, the Argonne interaction has been constructed to some degree with the idea that these terms should be small. In fact, the expectation value of the sum of these terms in light nuclei is only one to two MeV. Consequently, we solve exactly for a modified Argonne V8 (containing only the eight operators through $L \cdot S$) interaction which best approximates the full Argonne V14 model. This model reproduces the deuteron, the singlet S, and triplet P waves (with the exception of coupling to F waves) exactly. Perturbation theory is then used to estimate the difference between the V14 and V8 models, we find that this difference is small in the alpha particle. Finally, we note that this Argonne V8 model is somewhat different than that used in previous calculations.[13]

3. RESULTS

We will first concentrate on a new set of GFMC results obtained for the alpha particle with the AV8 nucleon-nucleon plus Urbana model 8 three-nucleon-interaction.[15] In order to demonstrate the convergence of the GFMC method, figure 1 shows the ground state energy plotted as a function of the total iteration time τ. At $\tau = 0$, the energy is equal to the variational result, and it quickly drops to the exact ground state energy. In fact, the plot covers only the initial part of the calculation, up to a total iteration time of 0.012 MeV^{-1}. The actual calculation includes 5 times as many iterations, the horizontal lines in the figure are statistical error bounds obtained by averaging the results between 0.024 and 0.060 MeV^{-1}. The convergence of the GFMC solution is determined by the accuracy of the trial wave function as well as the excitation structure of the nucleus.
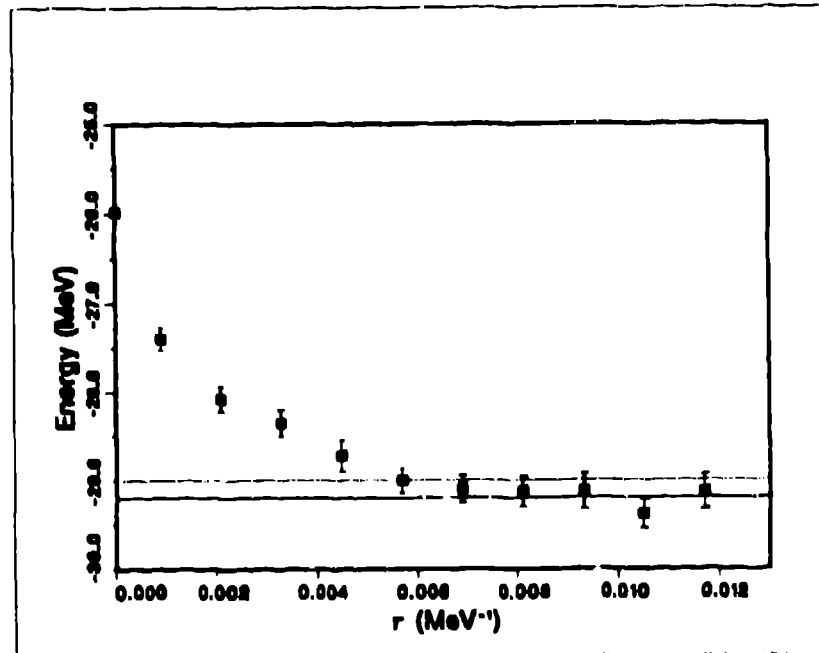

Figure 1) Alpha Particle Ground State Energy vs. iteration time τ.

The variational wave function used in this calculation was taken from reference 16, and was optimized for the Argonne V14 plus Urbana model 7 TNI. Consequently, it does not provide a very good estimate for the ground state energy with the model 8 TNI, which has a stronger repulsive component and a weaker two-pion-exchange term. However, the rms radius of this trial wave function is very near the exact result, hence it requires smaller extrapolations for the estimates of other properties. GFMC produces a wave function only in a statistical sense, and hence ground state energy expectation values other than the energy are extrapolated from 'mixed' and variational estimates via:

$$\langle \Psi_0 | H | \Psi_0 \rangle \approx \langle \Psi_T | H | \Psi_0 \rangle - \langle \Psi_T | O | \Psi_T \rangle. \tag{13}$$

The extrapolations required with the present variational wave function are generally quite small.

For the Argonne V8 plus TNI model 8 interaction, we obtain a ground state energy of -29.20 ± 0.15 MeV, approximately one MeV overbound compared to the experimental -28.3 MeV. The perturbative estimate of the difference between the Argonne V14 NN interaction and the V8 model is 0.9 MeV; yielding a total energy of -28.3 $\pm$ 0.2 MeV, in remarkably good agreement with the experimental result.

One should be somewhat cautious because of our use of perturbation theory in the difference between V14 and V8; but it appears that the same three body force can be used to produce very accurate binding energies for three and four body nuclei. The Urbana TNI model 8 has been chosen to provide a good fit to the triton binding energy,[17] Faddeev results give -8.46 compared to the experimental -8.48 MeV. The expectation value of the three nucleon interaction is a small fraction ($\leq 5\%$) of the total potential energy, so at this level there is no apparent reason to introduce four- or higher-body interaction terms. Other models (Reid, Nijmegen, ...) of the NN potential give a similar underbinding for the three- and four-body nuclei, hence it should be possible to fit the binding energies of these nuclei as well with an appropriate TNI model.

The most accurate variational calculations to date[12] give a binding energy approximately one MeV higher than this GFMC calculation. As always, the total binding energy includes a large cancellation between kinetic and potential terms, each of which are of the order of 100 MeV (Table 1). Therefore, although the TNI is a relatively small fraction of the two-nucleon interaction it is a significant part of the total binding energy, and accurate calculations are important when determining its effects. Also included in Table 1 are several other expectation values which, although not directly accessible experimentally, provide a useful guide to understanding these nuclei.

Of particular interest is the strong effect of the tensor interaction in the alpha particle. With the Argonne NN interaction, the tensor components contribute approximately 2/3 of the two-body potential energy in the alpha particle. Almost exactly the same fraction is found in Faddeev calculations of three-body nuclei and in cluster Monte Carlo calculations of

Table 1: Alpha Particle Expectation Values

Energy	-28.3	(0.2)
$\langle T \rangle$	109.3	(1.2)
$\langle V_{NN} \rangle$	-136.5	(1.5)
$\langle V_{coul} \rangle$	0.75	(0.01)
$\langle V_{3-s} \rangle$	5.0	(0.2)
$\langle V_{3-2\tau} \rangle$	-10.8	(0.2)
$\langle r_i^2 \rangle^{1/2}$	1.45	(0.01)

$^{16}O.^{18}$

Another measure of the strength of the tensor interaction is the D state probability in the four-nucleon ground state. With the Argonne plus Urbana model 8 TNI interaction, the D-state probability is 16%, other models range from 12 to 17 %. These probabilities are nearly consistent with what one would expect based upon the number of triplet pairs in the A=2, 3, and 4 body nuclei; a ratio of 1:1.5:3. In addition, the asymptotic D to S state ratio of the alpha particle wave function is in good agreement with experimental results.[19] The remainder of the wave function is dominated by the fully symmetric S-wave state, which has a probability of 82.8(0.2)%. In addition, there are small components of other symmetries, either S- or P-wave.
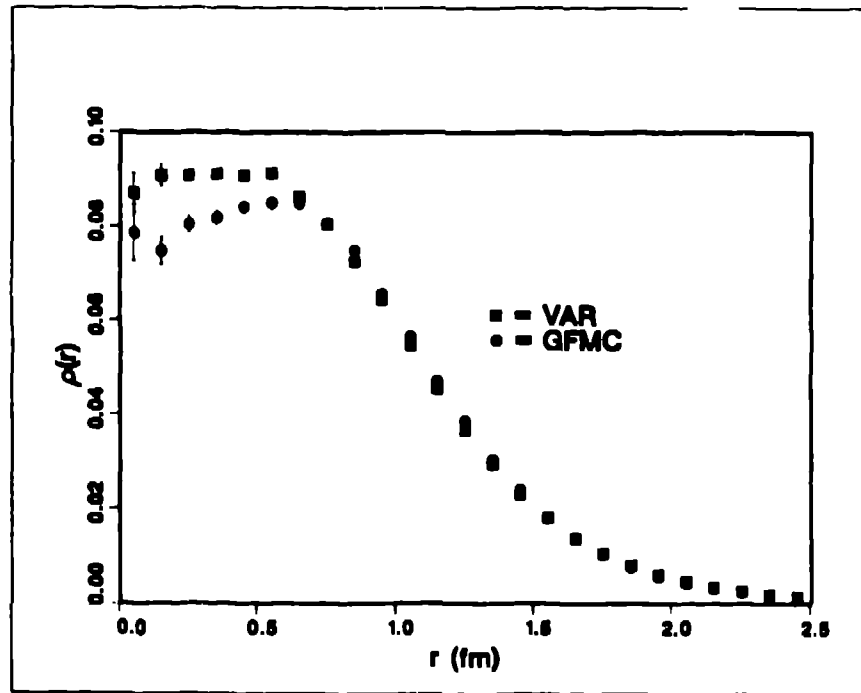

Figure 2) VMC and GFMC results for the proton density in the alpha particle.

We have also computed the proton density for both the variational and GFMC wave functions (Fig. 2). The most important difference is within 0.5 fm of the center-of-mass,

the GFMC wave function has a slight dip which does not appear in the variational results. This difference appears in only a very small fraction of the total volume of the nucleus due to the r^2 phase space factor at the origin. Consequently, the difference does not significantly affect the rms radius or the charge form factor at small momentum transfer. In the impulse approximation, the charge form factor can be obtained as the fourier transform of the one-body charge distribution.

In reality, though, the effects of two-body charge and current operators can be important even at relatively low momentum transfer. In order to obtain meaningful comparisons with experimental results, the effects of these two-nucleon operators must be incorporated into the calculations. Riska[20] has developed a method for constructing models of the exchange currents which satisfy the continuity equation:

$$\nabla \cdot \vec{j}_{ex} + i\,[V_{ij}, \rho] = 0. \tag{14}$$

This constraint is used to specify the 'model-independent' exchange currents. In addition, there are transverse pieces in the current (e.g. $N\Delta\gamma, \rho\pi\gamma$, and $\omega\pi\gamma$) which are not so constrained. The most important two-body terms in the current are due to the pion:

$$j_\pi(q) = -3i(\tau_i \times \tau_j)_z[\tilde{v}_\pi(k_j)\vec{\sigma}_i(\sigma_j \cdot k_j) - \tilde{v}_\pi(k_i)\vec{\sigma}_j(\sigma_i \cdot k_i) -$$
$$\frac{\vec{k}_i - \vec{k}_j}{k_i^2 - k_j^2}(\sigma_i \cdot k_i \sigma_j \cdot k_j)[\tilde{v}_\pi(k_j) - \tilde{v}_\pi(k_i)]]G_E^V(q), \tag{15}$$

where k_i is the momentum transfered to nucleon i and $\tilde{v}_\pi$ is the fourier transform of the terms in the interaction associated with the quantum numbers of exchanged pions. In the limit of point pions and nucleons,

$$\tilde{v}_\pi(k) \rightarrow \frac{1}{3}\frac{f_\pi^2}{m_\pi^2}\frac{1}{k^2 + m_\pi^2} \tag{16}$$

In fact, this method produces nearly point-like pion propagators with the Argonne interaction.

Using this method, Schiavilla and Riska have computed the magnetic form factors of ^{3}He and ^{3}H (Fig. 3), as well as the backward cross-section for the electrodisintegration of the deuteron. The different curves show both the impulse and impulse plus meson exchange current results, the contributions of the exchange currents are crucial to reproducing the experimental results, particularly the contribution of the isovector exchange current operators. Results with both the variational and Faddeev wave functions are also shown; there are some differences in the region of the diffraction minimum and beyond. Schiavilla and Riska have also calculated the backward electrodisintegration of the deuteron near threshold. This reaction is also very sensitive to the isovector exchange currents, and is well reproduced in the calculations, up to very high values of the momentum transfer.
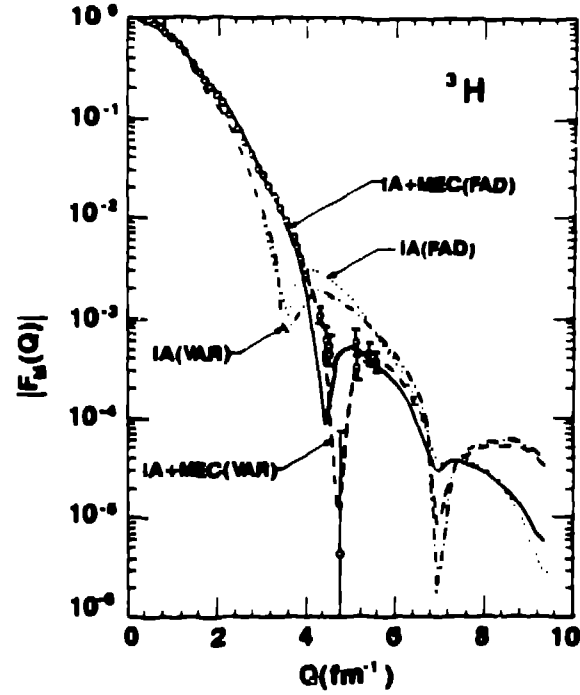
Figure 3a) Magnetic form factor of ^{3}H, from Schiavilla and Riska.[11] Impulse approximation (IA) results are shown along with the complete results (IA+MEC). Curves labeled FAD employ the exact Faddeev wave function, and variational results are labeled VAR.
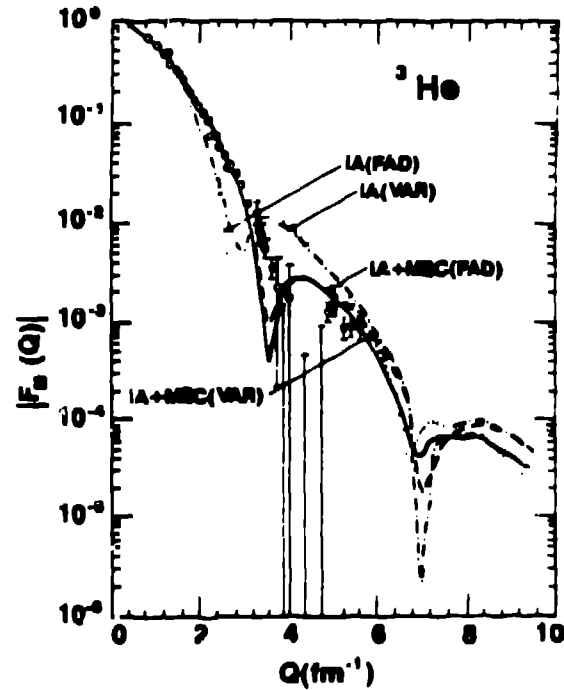

Figure 3b) Magnetic form factor of ^{3}He, as above.

They have also computed the charge form factors of the three-body nuclei,[21] and obtain good agreement with experimental results. The charge operators are more speculative since they involve relativistic corrections and are not constrained by the continuity equation.

However. in the alpha particle some of the uncertainties are decreased because of the isoscalar nature of the system. We have combined the following one-body charge operator:

$$\rho_1(q) = [1 - \frac{q^2}{8m^2}]\frac{1}{2}[G_E^S(q) + G_E^V(q)\tau_z]$$

$$-i\frac{\sigma \cdot q \times P}{8m^2}\frac{1}{2}\left\{[G_E^S(q) - 2G_M^S(q)] + [G_E^V(q) - 2G_M^V(q)]\tau_z\right\}. \qquad (17)$$

incorporating the Darwin-Foldy term and a small $L \cdot S$ correction, with a two-body charge operator due to pions:

$$\rho_\pi(q) = \frac{3}{2m}\left\{\left[F_1^S(q)\tau_i \cdot \tau_j + F_1^V(q)\tau_{jz}\right](\sigma_i \cdot q\sigma_j \cdot k_j)\tilde{v}_\pi(k_j)+\right.$$

$$\left.\left[F_1^S(q)\tau_i \cdot \tau_j + F_1^V(q)\tau_{iz}\right](\sigma_j \cdot q\sigma_i \cdot k_i)\tilde{v}_\pi(k_i)\right\} \qquad (18)$$

to calculate the charge form factor of the alpha particle. This form of charge operator was first considered by Kloet and Tjon in examining pion photoproduction.[22] We have also included the remaining terms ⌣ ˙ Schiavilla and Riska, but their effect is an order of magnitude smaller than the terms above up to a momentum transfer of ≈ 5.5 fm^{-1}. The contribution of the one-body and pion-exchange terms are shown in figure 4. As is apparent in the figure, the VMC and GFMC results give nearly identical results for the exchange currents. However, there is a significant difference between the one-body terms in the region of the second maximum. The form factor here is down by two orders of magnitude from that at the origin due to a sensitive cancellation in the fourier transform. Hence it is not surprising that the variational calculation differs significantly from the GFMC result in this region.
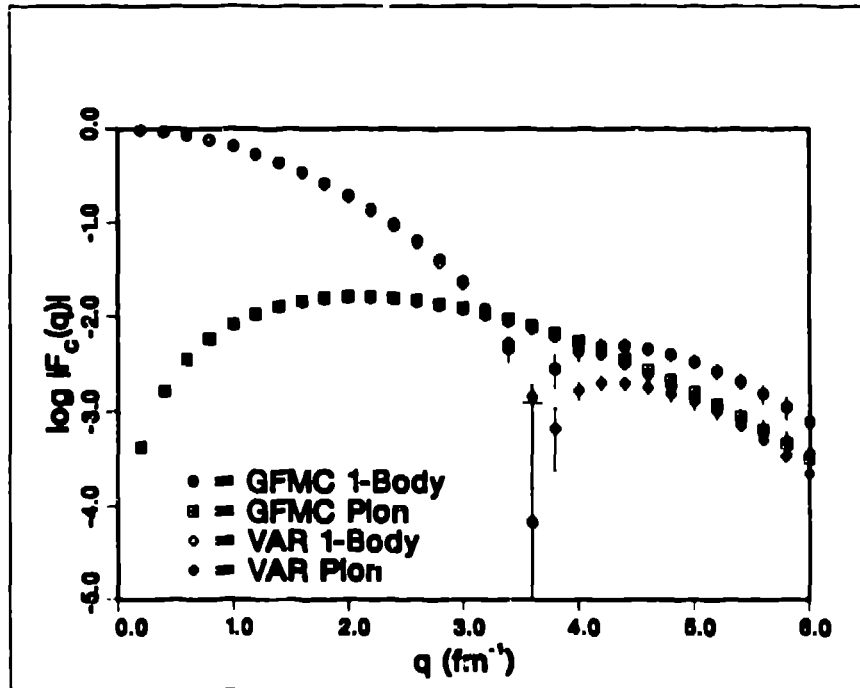

Figure 4) VMC and GFMC results for one-body and pion contributions to the alpha particle charge form factor.

The full calculations are compared to experimental results in figure 5. The GFMC calculation is in excellent agreement with experimental results up to a momentum transfer of ≈ 4.5 fm^{-1}. Beyond that point, the calculated form factor is significantly larger than experimental results. Nevertheless, the overall agreement is excellent, particularly at lower momentum transfers where one would expect the theory to work best.
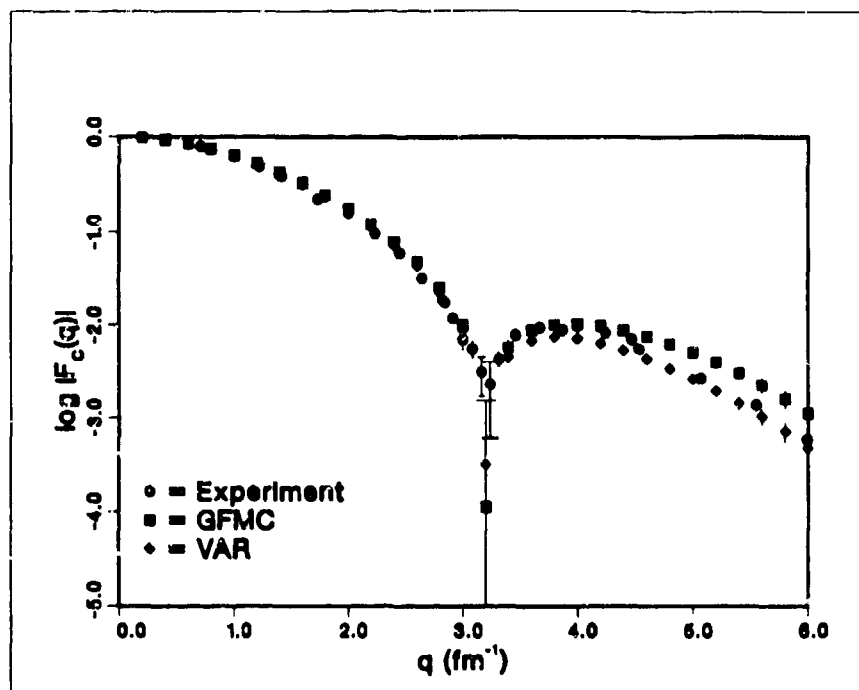

Figure 5) Alpha particle charge form factor, experimental and calculated.

A very important topic in nuclear physics is the experimental determination of the correlations between nucleons in nuclei. The Coulomb sum, measured in electron scattering experiments, is a valuable method for studying these correlations, and is defined as:

$$S = \frac{1}{Z} \int_{\omega_{el}^+}^{\infty} \frac{R_L(q,\omega)}{[G_E(q^2)]^2} d\omega, \tag{19}$$

where R_L is the longitudinal response of the nucleus and G_E is the proton form factor. The integral extends from energies just above elastic scattering to infinity, which allows us to use closure to calculate the Coulomb sum as a ground state expectation value.

$$S = \frac{1}{Z} \left[\langle 0 | \sum_{j=1}^{A} \rho_j^\dagger(q) \sum_{k=1}^{A} \rho_k(q) | 0 \rangle - \frac{[Z F_c(q^2)]^2}{[G_E(q^2)]^2} \right], \tag{20}$$

where

$$\rho_k(q) = \exp(iq \cdot r_k)[\frac{1 + \tau_{zk}}{2}] \tag{21}$$

if we ignore small neutron contributions (which are included in the calculations) and two-body terms. In this approximation, the Coulomb sum is simply:

$$S = 1 - Z\frac{[F_c(q^2)]^2}{[G_E(q^2)]^2} + \frac{1}{Z}\rho_{pp}(q), \tag{22}$$

where F_c is the charge form factor of the nucleus and $\rho_{pp}(q)$ is the fourier transform of the two-body distribution function integrated over the pair's center-of-mass.

The calculations are compared to experimental results in Figure 6. Two caveats should be noted concerning this comparison. First, the experimental results only extend to a finite energy, and consequently must be extrapolated to determine the full Coulomb sum. Schiavilla et al.[23,24] calculated the energy- and energy-squared weighted sum rules with a variational wave function; assumed a functional form for the response in the tail region, and fit this curve to the calculated moments. The contributions of the tail region in the experiment are given by the difference between the points labeled 'extr' and 'trunc'. The latter includes only the response up to the experimental limit. As shown in the figure, the VMC and GFMC curves are nearly identical, and both agree very well with the extrapolated results.
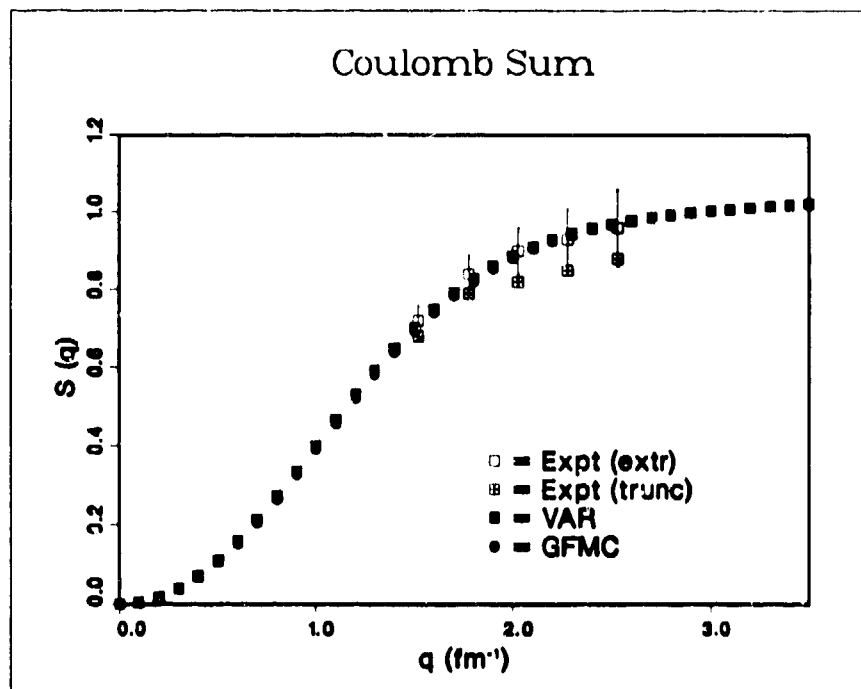

Figure 6) Coulomb sum in the alpha particle.

Beck[25] extracted $\rho_{pp}(q)$ from the experimental results in the three-nucleon system using a slightly different extrapolation technique and the theoretical results for the neutron contributions. Although the qualitative features of the experimental and theoretical curves are similar, the experimental $\rho_{pp}(q)$ is much higher beyond the first minimum. This would indicate even a stronger correlation in the protons than is present theoretically, but contributions of two-body operators to the Coulomb sum should be included before strong conclusions are drawn.

Once a consistent picture of the ground state properties of light nuclei has been obtained, there are two natural directions for future research. The first is calculations of the structure and properties of heavier nuclei, and the second is the study of dynamic properties. Heavier

nuclei are difficult to study because of the strong spin-isospin dependence in the interaction. Cluster Monte Carlo methods hold the most promise for calculations of many-body nuclei. In ^{16}O, Pieper et. al[10] found a ground state energy of -7.0 MeV per nucleon with the Argonne V14 plus TNI model 7 interaction, compared to the experimental binding energy of 8 MeV per nucleon. This form of variational wave function gives roughly the same binding per nucleon in the alpha particle.

Currently, this work is being extended to include more accurate forms for the wave function, including $L \cdot S$ and improved three-body correlations. Preliminary results indicate a significant increase in the binding energy, but some work remains to be done. Among the issues remaining to be resolved are a better understanding of the convergence of the cluster method for three-body and momentum-dependent two-body interactions. The Coulomb sum for ^{16}O has also been calculated, and is shown in Figure 7. There is no experimental data available for ^{16}O, so the data for ^{12}C is included in the figure. Also shown is a mean-field calculation in which it is assumed that there are no correlations between the nucleons. The data at small q demonstrate the presence of correlations, but there are large uncertainties at higher momentum transfer. It is interesting to note that the Coulomb sum cannot be smoothly extrapolated from light nuclei to nuclear matter.
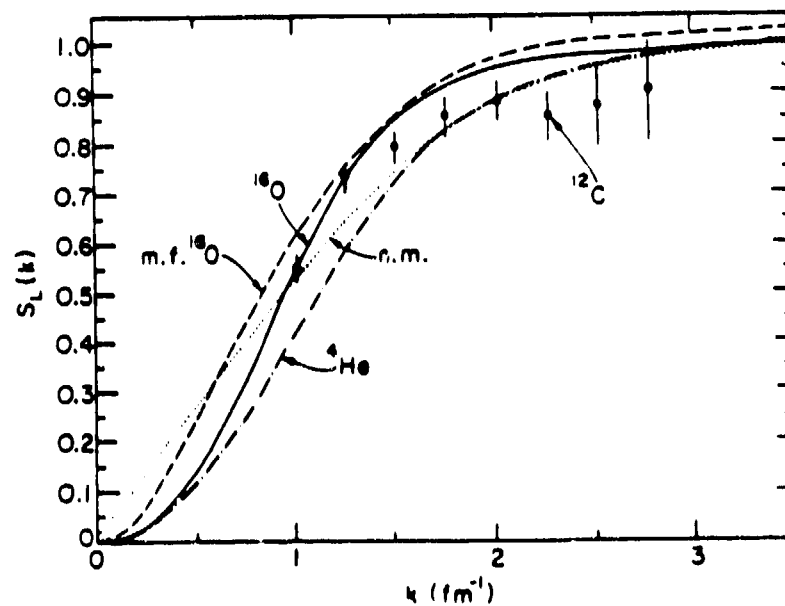

Figure 7) Coulomb sum in ^{16}O, from reference 10.

The other outstanding problem in the application of Monte Carlo methods to nuclear physics is the study of dynamic properties, a very ambitious goal. The primary successes to date have been in the study of low-energy scattering and electromagnetic transitions, and progress has been made in approximate treatments of dynamic response in electron

scattering.[26,27] I will concentrate on the former topic, and particularly upon the $n +$ ^{3}He $\to$ $\alpha + \gamma$ reaction.

Low energy scattering in a regime where only two-body breakup is energetically allowed can be treated with variational methods.[28] The basic idea is very similar to R-matrix approaches. In a one-channel problem the boundary condition is specified at a point beyond the interaction region, and then a variational search is performed to determine the energy eigenvalue appropriate to that boundary condition. This scheme can be generalized to multi-channel scattering processes, but requires a determination of the energies and relative amplitudes at the channel surfaces. The method's practicality depends upon the ability to diagonalize in a small basis (10 - 20 states) using Monte Carlo methods. Preliminary results on small problems indicate that this should be feasible, but multi-channel methods have not been tested on a realistic problem.

We have used this method to study the $n +$ ^{3}He $\to \alpha + \gamma$ reaction, which is dominated by one channel.[29] This reaction is of interest because of its possible relationship to the weak capture reaction in the four-nucleon system, a reaction which produces the highest end-point energy neutrinos from the sun. There have been speculations that these neutrinos could be measured separately in a future solar neutrino observatory. In the impulse approximation, the weak and electromagnetic capture are closely related.

Our calculations indicate, though, that this reaction is dominated by exchange currents. We obtain a strong-interaction scattering length of 3.5 $\pm$ 0.25 fm for the spin one n-^{3}He state, which agrees well with experimental estimates. Using this scattering wave function and a variational ^{4}He wave function, we find that only 10 % of the experimental value (60 μbarns) is obtained in the impulse approximation. The low value is to some extent understandable since the impulse cross section is precisely zero in the limit where there is no tensor force, and consequently a purely s-wave alpha particle.

Using the full exchange current models, we find a value of 110 μbarns for the cross section. Including only the better-constrained 'model-independent' terms in the exchange currents gives 70 μbarns, in much better agreement with the experiment. A similar result is obtained if we keep only the π exchange terms, as has been done in the three-body calculations of Friar, Gibson, and Payne; and use a cut-off of 5.8 π masses in the propagator. In this case we obtain a total cross section which agrees with the experimental value. Our results are quite sensitive to the scattering length, however, a decrease of 0.25 fm in the scattering length would increase the calculated cross sections considerably. Much work remains to be done in this area, as many important tests of strong-interactions and exchange currents are available.

4. Summary and Outlook

Monte Carlo methods provide a valuable tool for understanding the nuclear Hamiltonian and nuclear structure. They are also applicable to other areas of nuclear physics, for example quark-model physics. In this talk, I have emphasized applications to traditional models of nuclear physics and the successes of these models in describing properties of light nuclei. Realistic nucleon-nucleon interactions, combined with plausible three-nucleon-interaction models, give a good description of the binding energy of three- and four-body nuclei. Calculations employing these interactions demonstrate the very important role of the tensor force. When coupled with exchange currents, these 'traditional' models can also provide reasonable descriptions of the form factors of few-body nuclei.

Light nuclei are simple enough so that many calculations are practical, but complex enough to allow many interesting processes to be examined. Calculations of the Coulomb sum indicate the importance of nucleon-nucleon correlations, and low-energy reactions offer the opportunity for a wide variety of tests for the nuclear Hamiltonian and exchange current models. Many important challenges lie ahead in the 90's. Foremost among these are calculations of larger nuclei and development of new techniques for treating dynamics. Heavier nuclei offer the opportunity for studying the nuclear interaction in negative parity states and very neutron-rich nuclei, which are important astrophysically through their connection with neutron stars. A better understanding of current and future electron scattering experiments requires reliable calculations of the dynamic response of nuclei, perhaps the most challenging goal for the next decade.

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